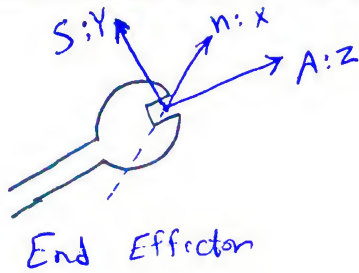


Robotics

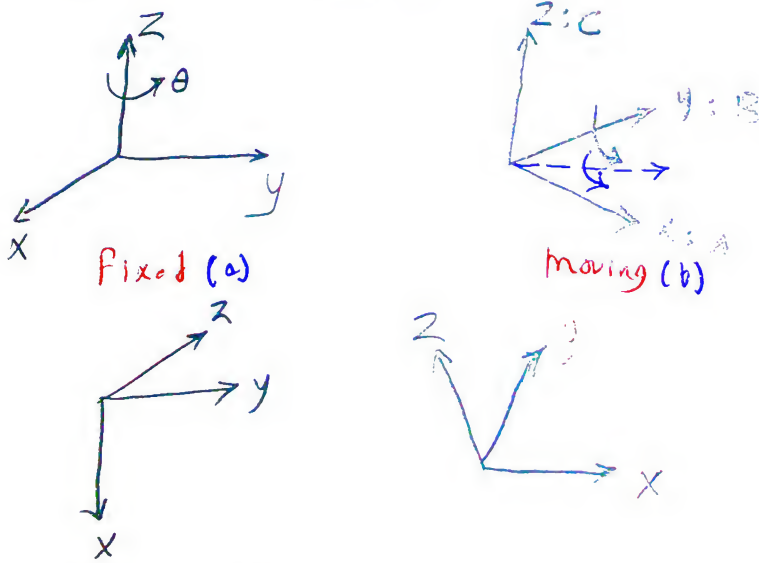
Section 4

Rotation Matrix

- 1- Orientation of Frame with another
- 2- Coordinates of point P with different frame
- 3- Rotation Operator



Composition and Rotation



around new y (C_1) Post multiply

around fixed y (C_2) Pre multiply

Successive rotation (Similarity Transformation)

$$R_{C_1} = \underset{\substack{\text{Fixed} \\ \text{Pre multiply}}}{R(z, \theta)} \mathbf{I} \underset{\substack{\text{Current} \\ \text{Post multiply}}}{R(y, \phi)}$$

$$R_{C_2} = R(y, \phi) R(z, \theta) \mathbf{I}$$

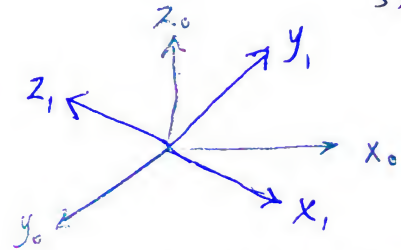
Example

- R : Specified by the following sequence
- 1- rotation of θ about current X -axis
 - 2- rotation of ϕ about current Z -axis
 - 3- rotation of α about fixed Z -axis
 - 4- rotation of β about current Y -axis
 - 5- rotation of δ about fixed X -axis

$$R = R(x, \delta) R(z, \alpha) \mathbf{I} R(x, \theta) R(z, \phi) R(y, \beta)$$

Parametrization of Rotations

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3 \times 3}$$



الدوران في الفراغ بشكل عام به تسع متغيرات
و يتم فيها تغيير مكان جميع المحاور
في الفراغ

Euler Angles (ZYZ) Representation \mathbf{I}
(ZBC)
($\theta \phi \psi$)

$$R = R(z, \theta) R(y, \phi) R(z, \psi)$$

$$= \begin{bmatrix} C\phi C\theta C\psi - S\phi S\psi & -C\phi S\theta & C\phi S\psi \\ S\phi C\theta C\psi + C\phi S\psi & -S\phi S\theta & S\phi S\psi \\ -S\phi C\psi & C\theta & C\psi \end{bmatrix}$$

Euler Angles (ZXZ)
(ZAC) representation II

$$R = R(z, \theta) R(x, \phi) R(z, \psi)$$

Roll - Pitch - Yaw
Z Y X

لتحويل السوريات إلى الفراغ

* Fixed elementary axis

- 1- ψ around x
- 2- ϕ around y
- 3- θ around z

$$R = R(z, \theta) R(y, \phi) R(x, \psi)$$